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ON THE NONUNIQUENESS AND INSTABILITY OF THE STATIONARY COMBUSTION MODES IN THE BOUNDARY LAYER, WITH STRONG INJECTIONS*

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A mathematical model given in /1/ describing approximately the heat and mass exchange between the inert thermostat and reacting gas flow in the presence of strong injection of a chemically frozen gas, is used to analyze the influence of the force of injection and other factors on the stability of combustion in the boundary layer. The boundaries of the regions of stability and uniqueness of the stationary combustion modes are constructed in the plane of the Damköhler numbers. At small Lewis numbers a region corresponding to the self-oscillatory combustion modes is discovered, and an estimate given for the influence exerted by various factors on the frequency and amplitude of the self-oscillations. It is established that a strong injection of a chemically frozen gas stabilises the combustion process, since the region of instability narrows appreciably with increasing force of injection. On the other hand, increase in the injection rate leads to appreciable increase in the size of the region of nonuniqueness of the stationary modes.

1. Formulation of the problem. We consider a flow of a reactive gas past the leading stagnation point of the inert body of revolution with constant surface temperature T_w, in the presence of a constant strong injection of a chemically frozen gas through the surface of the streamlined body. The injected gas initiates in the boundary layer a combustion reaction, in which either the components of both incoming and injected stream, or only the components of the incoming stream) take part. We assume that the reaction is a first order reaction, and its rate depends on the temperature in accordance with the Arrhenius law, and on the concentration of a single, rate determining component of the gaseous mixture. The gaseous mixture is assumed by be effectively binary /2/, the radiation can be neglected, the Prandtl and Schmidt numbers and the product of density and viscosity are all constant, the specific heat capacities of different components are constant and equal to eath other, while the equations of motion and continuity are quasistationary. Our aim is to investigate the possible modes of the combustion reaction.

Under the above assumptions the mathematically formulated problem is reduced to the qualitative investigation of a boundary value problem /1/ for a system of two quasilinear parabolic equations for the temperature and concentration of the rate determining component, and a single ordinary differential equation for the stream function. At present the qualitative theory of the boundary value problems of this type lacsk an effective method of investigation. For this reason the author gave in /1/ an approximate method of qualitative investigation of the boundary value problem in question, based on its reduction to a dynamic second order system. The reduction is based on the hypothesis of strong injection /3–5/ and on the procedure described in /6/. In the final count the problem reduces, in accordance with /1/, to a qualitative analysis of the following dynamic system describing in approximate manner the combustion of gas near the contact surface:

$$\frac{dC_\Delta}{d\tau} = a(C_e - C_\Delta) - L^{1/2}b(C_\Delta - C_w) - C_1 C_\Delta \exp\left[\bar{E}\left(1 - \frac{1}{\bar{T}_\Delta}\right)\right] \equiv P(C_\Delta, \bar{T}_\Delta) \tag{1.1}$$

$$\frac{d\bar{T}_\Delta}{d\tau} = a(1 - \bar{T}_\Delta) - b(\bar{T}_\Delta - \bar{T}_w) + C_1 C_2 C_\Delta \exp\left[\bar{E}\left(1 - \frac{1}{\bar{T}_\Delta}\right)\right] \equiv Q(C_\Delta, \bar{T}_\Delta)$$

$$\tau = \frac{t}{t_a}, \quad t_a = \frac{1}{\beta_x}, \quad \bar{T} = \frac{T}{T_e}, \quad \bar{E} = \frac{E}{RT_e}, \quad a = \sqrt{2\beta_1 \frac{\rho_e}{\rho_w}}$$

$$b = \frac{1}{(-2f_w)}\sqrt{\frac{\pi a^3}{2Pr}}, \quad \frac{\rho_e}{\rho_w} = \frac{T_w M_e}{T_e M_w}, \quad C_1 = k_0 \exp(-\bar{E})/(2\beta_x)$$

$$\beta_x = \left(\frac{du_e}{dx}\right)_{x=0}, \quad C_2 = \frac{q}{c_p T_e}, \quad L = \frac{Pr}{Sc}, \quad Pr = \frac{\mu c_p}{\lambda}, \quad Sc = \frac{\mu}{\rho D}$$

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Here τ is the dimensionless time, t is time, t_a is the characteristic aerodynamic time; f and $\bar{T}$ are the dimensionless stream function and temperature, C is the mass concentration of the rate determining component of the gaseous mixture, T is temperature, $\bar{E}$ is the dimensionless activation energy; a and b are dimensionless parameters; C_1 and C_2 are the first and second Damköhler numbers; L, Pr and Sc are the Lewis, Prandtl and Schmidt numbers, β_1 is a dimensionless parameter characterizing the geometry of blunting of the body of revolution (for the spherical and cylindrical blunting we have $\beta_1 = 0.5$ and $\beta_1 = 1$ respectively), β_x is the velocity gradient of the unperturbed flow at the stagnation point, x is a rectangular coordinate directed along the generatrix of the body of revolution, u is the x-component of the velocity of gas; E, q and k_0 are the activation energy, thermal effect and the preexponent of the reaction, R is the universal gas constant; ρ and μ are the density and dynamic viscosity of the gas, M is the molecular weight of the mixture, c_p is heat capacity at constant pressure, λ is the heat conductivity coefficient, D is the effective coefficient of diffusion; the indices e, w and Δ denote the gas parameters at the outside of the boundary layer, at the surface of the streamlined body, and at the contact surface respectively.

Notes. 1°. The mathematical model (1.1) is suitable, generally speaking, for describing the combustion near the contact surface in the case when the reagents have not previously been mixed, as well as in the case when all reagents are present in the incoming stream and the zone of combustion is displaced from the contact surface into the depth of boundary layer. In the latter case however, the asymptotic estimate of the improper integrals carried out in the course of deriving (1.1) can, according to /6/, lead to appreciable errors, and this reduces the accuracy of the model.

2°. The accuracy of the mathematic model (1.1) increases with increasing injection, since in deriving the system (1.1) we used the expansions of the functions f, $\bar{T}$ and C near the streamlined surface, based on the hypothesis of strong injection. The injection is assumed to be strong /3–5/ if the injection parameter $a = -f_w \geqslant 2$. The accuracy of the model (1.1) diminishes under moderate injection ($0 < a < 2$). The expansion mentioned above and hence the system (1.1) become unsuitable in the case when the injected gas is not chemically frozen and can enter into reaction at once near the streamlined surface. In the latter case it is expedient to use the second dynamic system described in /1/.

3°. The need for solving the problem in question arises from the fact that in the course of numerical investigation of a layer which is not in chemical equilibrium one encounters such anomalous phenomena /7/ as the nonuniqueness and instability of the stationary modes. This is confirmed by e.g., the numerical experiments /8–10/ carried out for the case of hot gaseous mixtures flowing past heated inert thermostats, without injection /8,9/ and with moderate injection of an inert gas /10/. However, such investigations have not been carried out for the case of strong injection.

2. Analysis of the dynamic system (1.1) and physical interpretation of the results. The stationary combustion modes have the corresponding equilibrium states of the dynamic system (1.1). The coordinates C_Δ°, $\bar{T}_\Delta^\circ$ of the quilibrium states are obtained on the phase plane C_Δ, $\bar{T}_\Delta$ from the system of equations $P(C_\Delta^\circ, \bar{T}_\Delta^\circ) = 0$, $Q(C_\Delta^\circ, \bar{T}_\Delta^\circ) = 0$ which yields

$$C_\Delta^\circ = \frac{aC_e + L^{1/3}bC_w}{a + L^{1/3}b} - \frac{\varphi(\bar{T}_\Delta^\circ)}{C_2(a + L^{1/3}b)} \tag{2.1}$$
$$\varphi(\bar{T}_\Delta^\circ) = (a+b)\bar{T}_\Delta^\circ - (a + b\bar{T}_w)$$

$$C_1[C_2(aC_e - L^{1/3}aC_w) - \varphi(\bar{T}_\Delta^\circ)]\exp\left[\bar{E}\left(1 - \frac{1}{\bar{T}_\Delta^\circ}\right)\right] = (a + L^{1/3}b)\varphi(\bar{T}_\Delta^\circ) \tag{2.2}$$

The number of the states of equilibrium is equal to the number of roots of equation (2.2). In the limiting cases when the boundary layer is chemically frozen ($C_1 = 0$) or in chemical equilibrium ($C_1 \to \infty$), the equation has the following obvious unique solutions:

$$\bar{T}_\Delta^\circ = \bar{T}_{\Delta 0}^\circ \equiv \frac{a + b\bar{T}_w}{a+b}, \quad C_\Delta^\circ = C_{\Delta 0}^\circ \equiv \frac{aC_e + L^{1/3}bC_w}{a + L^{1/3}b} \quad (C_1 = 0)$$
$$\bar{T}_\Delta^\circ = \bar{T}_{\Delta\infty}^\circ \equiv \frac{a + b\bar{T}_w}{a+b} + C_2\frac{aC_e + L^{1/3}bC_w}{a+b}, \quad C_\Delta^\circ = C_{\Delta\infty}^\circ = 0 \quad (C_1 \to \infty)$$

In all intermediate cases when the boundary layer is not in chemical equilibrium ($0 < C_1 < \infty$) the problem of determining the number of the equilibrium states and their positions on the phase plane is complicated by the fact that the transcendental equation (2.2) cannot be solved by analytic methods. We can however, provide a rough estimate for the position of the equilibrium states. Indeed, when $\bar{T}_\Delta^\circ < \bar{T}_{\Delta 0}^\circ$ and $\bar{T}_\Delta^\circ > \bar{T}_{\Delta\infty}^\circ$, then one part of (2.2) will be positive

and the other negative since $\varphi < 0$ when $\bar{T}_\Delta^\circ < \bar{T}_{\Delta 0}^\circ$ and $\varphi > C_2 (aC_e + L^{1/2} bC_w)$ when $\bar{T}_\Delta^\circ > \bar{T}_{\Delta\infty}$. Therefore from (2.1) and (2.2) we obtain, for $0 < C_1 < \infty$,

$$\bar{T}_{\Delta 0}^\circ < \bar{T}_\Delta^\circ < \bar{T}_{\Delta\infty}^\circ, \quad 0 < C_\Delta^\circ < C_{\Delta 0}^\circ \tag{2.3}$$

To obtain a more accurate estimate of the position of the equilibrium states, equation (2.2) must be solved numerically.

Let us turn our attention to the problem of determining the number of roots of (2.2) at $0 < C_1 < \infty$. First we transform it to the form

$$\left[\frac{C_2 E (aC_e + L^{1/2} bC_w)}{(a+b)\Theta_\Delta^\circ - b\Theta_w} - 1\right] \exp \frac{\Theta_\Delta^\circ}{1 + \Theta_\Delta^\circ / E} = \frac{a + L^{1/2} b}{C_1} \tag{2.4}$$

$$\Theta_\Delta^\circ = (\bar{T}_\Delta^\circ - 1) E = \frac{(T_\Delta^\circ - T_e) E}{R T_e^2}, \quad \Theta_w = (\bar{T}_w - 1) E = \frac{(T_w - T_e) E}{R T_e^2}$$

where Θ_Δ°, Θ_w denote the dimensionless stationary temperature of the contact surface and the temperature of the streamlined surface. Introducing the additional notation

$$B = C_2 E (aC_e + L^{1/2} bC_w), \quad \delta = a + b, \quad \varepsilon = b\Theta_w, \quad \nu = E, \quad \xi = \frac{a + L^{1/2} b}{C_1}$$

we find that equation (2.4) for Θ_Δ° coincides with equation (10) of /11/, and the results of /11/ lead to the following conclusions:

1°. For any $0 < C_1 < \infty$ the equation (2.4) has at least one solution Θ_Δ° and, according to (2.3),

$$\frac{b\Theta_w}{a+b} < \Theta_\Delta^\circ < \frac{b\Theta_w}{a+b} + C_2 E \frac{aC_e + L^{1/2} bC_w}{a+b}$$

2°. When $0 < C_1 < \infty$, equation (2.4) has a unique solution provided that

$$C_2 \leqslant C_2^* \equiv \frac{4(a + b\bar{T}_w)^2}{(aC_e + L^{1/2} bC_w)[(E-4)a + (E - 4\bar{T}_w) b]} \tag{2.5}$$

3°. If $C_2 > C_2^*$, then bifurcation values $C_{1,1}$ and $C_{1,2}$ of the parameter C_1 exist, on traversing these values the number of solutions of (2.4) changes. When $0 < C_1 < C_{1,1}$ and $C_1 > C_{1,2}$, the solution is unique, while three different solutions exist for $C_{1,1} < C_1 < C_{1,2}$. Thus for any $0 < C_1 < \infty$ the dynamic system (1.1) has at least one state of equilibrium which becomes unique when $C_2 \leqslant C_2^*$. The necessary and sufficient conditions for the nonuniqueness of the equilibrium states have the form

$$C_2 > C_2^*, \quad C_{1,1} < C_1 < C_{1,2} \tag{2.6}$$

The results obtained agree with those obtained by analyzing (1.1) in /1/, and supplement them. According to /1/ the number, type and stability of the equilibrium states of system (1.1) are determined by the signes of the quantities

$$\Delta_1 = C_1 \exp\left[E\left(1 - \frac{1}{\bar{T}_\Delta^\circ}\right)\right][a + b - (a + L^{1/2} b) C_2 E C_\Delta^\circ (\bar{T}_\Delta^\circ)^{-2}] + (a + L^{1/2} b)(a + b)$$

$$\sigma_1 = C_1 \exp\left[E\left(1 - \frac{1}{\bar{T}_\Delta^\circ}\right)\right][C_2 E C_\Delta^\circ (\bar{T}_\Delta^\circ)^{-2} - 1] - 2a - (1 + L^{1/2}) b$$

If the conditions (2.6) do not hold, then the system (1.1) has a unique equilibrium state which is an antisaddle ($\Delta_1 > 0$), stable at $\sigma_1 < 0$ and unstable at $\sigma_1 > 0$. When conditions (2.6) hold, the system (1.1) has three equilibrium states, one of which must be a saddle /1/ ($\Delta_1 < 0$), i.e. an absolutely unstable equilibrium state, and the other two are antisaddles (nodes or foci) the stability of which is determined by the sign of σ_1. The unstable antisaddles ($\Delta_1 > 0$, $\sigma_1 > 0$) can exit only when $L < 1$ /1/.

Combining $\Delta_1 = 0$ with the formulas (2.1) and (2.2) we obtain the parametric equations for the boundary of the reigon (2.6) of nonuniqueness of equilibrium states on the C_1, C_2 - parameter plane. The boundary represents, at the same time, a boundary of the region of monotonous or quasistationary instability, and the equations are

$$C_1 = \frac{a + L^{1/2} b}{(a+b)(\bar{T}_\Delta^\circ)^2} [E\varphi(\bar{T}_\Delta^\circ) - (a+b)(\bar{T}_\Delta^\circ)^2] \exp\left[E\left(\frac{1}{\bar{T}_\Delta^\circ} - 1\right)\right] \tag{2.7}$$

$$C_2 = \frac{E\varphi^2(\bar{T}_\Delta^\circ)}{aC_e + L^{1/2} bC_w} [E\varphi(\bar{T}_\Delta^\circ) - (a+b)(\bar{T}_\Delta^\circ)^2]^{-1}$$

If we construct the curve (2.7) on the C_1, C_2-parameter plane with the values of the remaining parameters fixed, then the determination of the bifurcation values $C_{1,1}$ and $C_{1,2}$ is reduced to direct read-off from the curve (2.7) at the given value of C_2. Similarly, combining the relation $\sigma_1 = 0$ with the formulas (2.1) and (2.2) we obtain the following parametric equations for the boundary of instability of the antisaddles, or of the transitional oscillatory instability:

$$C_1 = [\bar{E}\varphi(\bar{T}_\Delta^\circ)(\bar{T}_\Delta^\circ)^{-2} - 2a - (1 + L^{1/3})b]\exp\left[\bar{E}\left(\frac{1}{\bar{T}_\Delta^\circ} - 1\right)\right] \tag{2.8}$$

$$C_2 = \frac{\varphi(\bar{T}_\Delta^\circ)[\bar{E}\varphi(\bar{T}_\Delta^\circ) - (a+b)(\bar{T}_\Delta^\circ)^2]}{(aC_e + L^{1/3}bC_w)\{\bar{E}\varphi(\bar{T}_\Delta^\circ) - [2a + (1+L^{1/3})b](\bar{T}_\Delta^\circ)^2\}}$$

Here and in (2.7) the function $\varphi(\bar{T}_\Delta^\circ)$ is determined from (2.1), and the quantity $\bar{T}_\Delta^\circ$ serves as a parameter varying, by virtue of (2.3), over the interval $(\bar{T}_{\Delta 0}^\circ, \bar{T}_{\Delta\infty}^\circ)$.

The changes in the relative position of the curves (2.7) and (2.8) on the C_1, C_2 parameter plane when the remaining parameters vary, is of considerable interest, since it is their distribution that determines, in the final count, the possible combustion modes /11/. The position of the curves (2.7) and (2.8) on the C_1, C_2-plane depends on seven dimensionless parameters $a, b, \bar{T}_w, \bar{E}, L, C_w, C_e$. For the further analysis it is expedient to replace the parameters C_1, C_2 and C_w by new parameters

$$\bar{C}_1 = C_1 \exp\bar{E}, \quad \bar{C}_2 = C_2 C_e = \frac{qC_e}{c_p\bar{T}_e}, \quad \bar{C}_w = \frac{C_w}{C_e} \tag{2.9}$$

Here we assume that $C_e \neq 0$, i.e. the rate determining component is present in the incoming stream. Taking (2.9) into account we conclude, that the behavior of the curves (2.7) and (2.8) on the $\bar{C}_1, \bar{C}_2$-plane will depend only on the following six dimensionless parameters: $a, b, \bar{T}_w, \bar{E}, L, \bar{C}_w$.

Let us see how the above parameters affect the position of the boundaries of nonuniqueness and transitional instability in the presence of moderate and strong injection.

Case of moderate injection $(0 < -f_w < 2)$. Irrespective of the fact that the accuracy of the model (1.1) is not great in the case of moderate injection by virtue of the note 1°, it is this case that ought to be considered first since it is only at $0 < -f_w < 2$ that we can compare the results obtained with the estimates of the values of $\bar{C}_{1,1}$ and $\bar{C}_{1,2}$ given in /9,10/. We write, in accordance with /10/,

$$\beta_1 = 0.5, \quad \bar{M}_w = M_w/M_e = 1, \quad \bar{T}_w = 3, \quad Pr = 0.74$$
$$\bar{C}_w = 0, \quad f_w = -0.5, \quad \bar{E} = 67.1366, \quad L = 1$$

For the above selected values of the parameters we have $a = \sqrt{3}$, $b \approx 3.321$ and the curves (2.7) and (2.8) have the form shown in Fig.1 where the solid curve 1 represents the boundary of nonuniqueness $(\Delta_1 = 0)$ and the dashed curve the boundary of the region of instability of the antisaddles $(\sigma_1 = 0)$. The region in which the stationary solution of the system (1.1) are unique is situated outside the solid, wedge-shaped curve, and the region of stability of the antisaddles lies outside the dashed curve. Analyzing Fig.1 we find that when $\bar{C}_2 \leqslant \bar{C}_2^* \approx 1.08$, then the system (1.1) has a unique state of equilibrium, namely a stable antisaddle $(\Delta_1 > 0, \sigma_1 < 0)$ for any $\bar{C}_1$ which has a corresponding, in the physical sense, stable stationary reaction mode. For any $\bar{C}_2 > \bar{C}_2^*$ there exists a range $(\bar{C}_{1,1}, \bar{C}_{1,2})$ of values of the parameter $\bar{C}_1$ within which the system (1.1) has, according to what was said before, three equilibrium states one of which is a saddle not corresponding to any, physically feasible stationary mode, and the other two are stable antisaddles irrespective of the position of the point $(\bar{C}_1, \bar{C}_2)$ relative to the dashed curve $\sigma_1 = 0$. In fact, when $L = 1$ then the condition that $\sigma_1 \geqslant 0$ can hold, according to /1/, only in the case of a saddle $(\Delta_1 < 0)$. Therefore the dashed curve in Fig.1 is of formal character only and does not represent a boundary of the region of instability.

The authors of /9,10/ give an estimate of the interval over which the number of the stationary modes can vary for $\bar{C}_2 \sim 6.44518$. Using the graph of (2.7) in Fig.1 we find, that for the given $\bar{C}_2$ $\bar{C}_{1,1} = 2.5\cdot10^8, \bar{C}_{1,2} = 3\cdot10^{11}$. The values of $\bar{C}_{1,1}$ and $\bar{C}_{1,2}$ obtained have the same order of magnitude as those of /9,10/. A better agreement could not be expected, since on one hand the accuracy of the model (1.1) is poor at $f_w = -0.5$, on the other hand some additional assumptions were made in /9,10/, e.g. in solving the equation numerically for the dimensionless stream function, the flow was assumed incompressible.

Using the results of /9,10/ we can furnish the bifurcation values $\bar{C}_{1,1}$ and $\bar{C}_{1,2}$ with a clear physical meaning: $\bar{C}_{1,1}$ is associated with extinguishing of the mixture and $\bar{C}_{1,2}$ with its ignition, and the interval $(\bar{C}_{1,1}, \bar{C}_{1,2})$ of nonuniquenss of the equilibrium states of the system

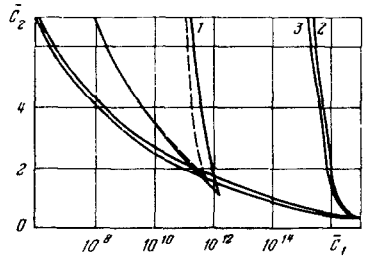

Fig.1

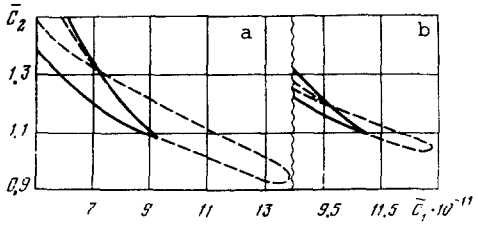

Fig.2

(1.1) with the transition zone situated between the chemically frozen and chemical equilibrium states of the boundary layer. The behavior of the curves $\Delta_1 = 0$ and $\sigma_1 = 0$ shown in Fig.1 remains unchanged at $L > 1$. The curve $\sigma_1 = 0$ at $L \geqslant 1$ has a wedge-like shape and lies wholly in the region of nonuniqueness ($\Delta_1 < 0$). Both antisaddles of (1.1) are stable when $\bar{C}_{1,1} < \bar{C}_1 < \bar{C}_{1,2}$. Establishing one or another stationary solution depends on the initial conditions. On the other hand, when $L < 1$, a decrease in the value of L is accompanied by an essential change in the behavior of the curve $\sigma_1 = 0$, namely, the curve forms a loop a part of which appears in the region of uniqueness. The situation is shown in Fig.2 where the solid and dashed lines represent the boundaries of the regions of nonuniquenesses and instability respectively, for $L = 0.01$ (Fig.2a) and $L = 0.05$ (Fig.2b) with the remaining parameters unchanged. It is clear that the loop increases with decreasing L. If the point $(\bar{C}_1, \bar{C}_2)$ falls within the region lying inside the loop but outside the solid wedge, then the system (1.1) will have a unique unstable equilibrium state ($\Delta_1 > 0$, $\sigma_1 > 0$), surrounded by a stable limit cycle /1/, and for these values of $\bar{C}_1$ and $\bar{C}_2$ we have a self-oscillatory mode of combustion.

The self-oscillations are depicted in Fig.3 showing the graphs of the relative concentration of the rate determining component $\bar{C}_\Delta = C_\Delta / C_e$ and dimensionless temperature $\Theta_\Delta = (\bar{T}_\Delta - 1)\bar{E}$ at the contact surface versus and dimensionless time τ. The graphs are based on the data obtained by numerical integration of the system (1.1) at

$$\beta_1 = 0.5,\ \bar{M}_w^{-1} = 2.389,\ T_w = 1,\ Pr = 0.74,\ \bar{C}_w = 0,\ f_w = -0.7,\ \bar{E} = 100,\ C_1 = 0.2,\ \bar{C}_2 = 0.1$$

for the case $L = 10^{-4}$ (solid lines) and $L = 2.5 \cdot 10^{-3}$ (dashed lines). The period of self-oscillations increases with increasing Lewis number, while $L = 0.01$ the self-oscillations vanish and a stationary mode is established after a time. A similar pattern obtains on reducing the parameter $\bar{E}$. For $L = 10^{-4}$ and $\bar{E} = 50$ the period of self-oscillations is 1.5 times greater than at $\bar{E} = 100$ and the self-oscillations disappear when $\bar{E} = 30$.

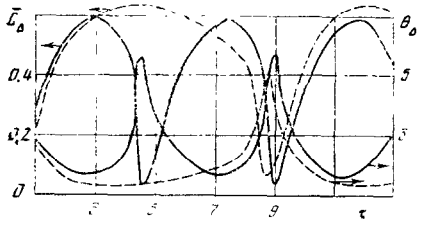

Fig.3

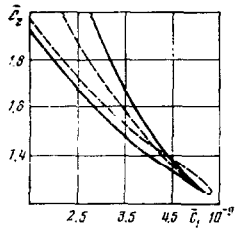

Fig.4

The self-oscillations shown above as well as the thermokinetic oscillations of the temperature and concentration fields, which were studied in /8,12/, are relaxation-type oscillations and are caused by the fact that at $L \ll 1$ the reaction time t_p is shorter than the thermal relaxation time t_e or diffusion time t_d. In fact, according to the graphs in Fig.3 we have, for $L = 10^{-4}$ $t_p/t_e \approx 0.54$, $t_p/t_d \approx 0.45$, while for $L = 2.5 \cdot 10^{-3}$ $t_p/t_e \approx 1.1$, $t_p/t_d \approx 0.79$. With further increase in the value of L the reaction time t_p becomes longer than either t_e or t_d, and the self-oscillations cease.

Thus the decrease in the Lewis number L and increase in the dimensionless energy of activation $\bar{E}$ leads to destabilisation of the combustion process in the boundary layer. At small L and large $\bar{E}$ a basically nonstationary combustion process, i.e. a self-oscillatory process is possible. The most drastic nonstationary effects appear when $L \to 0$ and $\bar{E} \to \infty$, therefore a study of the dynamic system (1.1) in the limit case when $L = 0$ and $\bar{E} \to \infty$ is worthwhile. It is interesting that when $L = 0$ the system (1.1) coincides, with the accuracy of up to the notation, with the dynamic system /11/ describing the nonstationary heat and mass exchange in a tubular chemical reactor with perfect mixing. When $L = 0$ and $\bar{E} \to \infty$, then the relative distribution of the uniqueness and stability boundaries in the C_1/a, $\bar{C}_2$ -plane, which determines

the behavior of the solutions of (1.1), depends on two parameters $\bar{T}_w$ and $\bar{b} = b/a$ only. When $\bar{T}_w = 1$, then according to /11/ the distribution of the curves $\Delta_1 = 0$ and $\sigma_1 = 0$ has the characteristic form shown in Fig.1, provided that $\bar{b} < \bar{b}^* = 0.7$. If on the other hand $\bar{b} \geqslant \bar{b}^*$, then the curve $\sigma_1 = 0$ forms a loop and dynamic system (1.1) has a manifold of possible solutions for various values of C_1, C_2 described in /11,13,14/. In particular, self-oscillatory modes become possible when $\bar{b} \geqslant \bar{b}^*$. When $\bar{b}$ increases, so does the size of the loop, and a part of the loop appears within the region of uniqueness. When $\bar{T}_w \neq 1$, changes in the value of $\bar{b}$ causes analogous changes in the behavior of the curves $\Delta_1 = 0$ and $\sigma_1 = 0$, with $\bar{b}^*$ assuming a different numerical value.

Thus the combustion process becomes destabilised with increasing parameter $\bar{b}$. Since

$$\bar{b} = \sqrt{\frac{\pi}{2Pr}} \sqrt[4]{2\beta_1 \frac{\bar{T}_w M_e}{T_e M_w}} (-2f_w)^{-1}$$

it follows that $\bar{b}$ can increase either because the streamlined surface becomes hotter (increase in $\bar{T}_w$), injection is reduced (reduction in $\alpha = -f_w$), or a lighter gas is injected (reduction in $\bar{M}_w$). Conversely, cooling the streamlined surface, increased injection and injection of a heavier gas, all lead to decrease in the value of $\bar{b}$ and hence to stabilisation of the combustion process. It must be noted that the parameter $\bar{b}$ represents the adiabatic index of the combustion process. Indeed, when $\bar{b} = 0$, we have $T^\circ_{\Delta 0} = T_e$, $T^\circ_{\Delta\infty} = T_e + qC_e/c_p = T_b$ where T_b denotes a so-called maximum adiabatic temperature of combustion. The case $\bar{b} = 0$ occurs when $\alpha = -f_w \to \infty$, in which case the contact surface coordinate $\Delta = -2f_w/a \to \infty$ becomes vanishingly small by virtue of the results of /1/ and the influence of the streamlined surface on the heat and mass exchange within the combustion zone. Since $\bar{b} = k\Delta^{-1}$ where $k = (\pi a/(2Pr))^{1/2}$, the influence mentioned above incrases with increasing $\bar{b}$ and the combustion process becomes non-adiabatic.

Case of strong injection $(-f_w \geqslant 2)$. In the case of strong injection we have, for $\beta_1 = 0.5$ and $Pr = 0.74$,

$$\bar{b} \leqslant \bar{b}_* \approx 0.364 \left(\frac{\rho_e}{\rho_w}\right)^{1/4} = 0.364 \left(\frac{\bar{T}_w}{\bar{M}_w}\right)^{1/4}$$

If $\bar{T}_w = 1$ and $\bar{M}_w = 1$, then $\bar{b}_* \approx 0.364 < \bar{b}^* = 0.7$. This implies that, even in the limit case when $L = 0$ and $E \to \infty$, the curve $\sigma_1 = 0$ forms no loop and lies wholly in the region of nonuniqueness in the C_1, C_2-plane. The pattern remains the same when $L > 0$ and $E < \infty$, therefore it follows that in the case of strong injection with $\bar{T}_w = 1$ and $\bar{M}_w = 1$ the combustion will always proceed into the stationary mode. When $\bar{T}_w$ increases and $\bar{M}_w$ decreases, loss of stability in the stationary modes becomes possible. The loss is connected with the appearance of a loop in the curve $\sigma_1 = 0$ at $\bar{b} = \bar{b}^*$. When $\bar{T}_w = 1$, this can happen only when $\bar{M}_w < \bar{M}_w^* \approx 0.073$, since by virtue of /11/ when $\bar{T}_w = 1, L = 0$ and $E \to \infty$, then $\bar{b}^* = 0.7$ and the inequality $\bar{b}_* > \bar{b}^*$ leads to the condition $\bar{M}_w < (0.52)^4$. When $\bar{T}_w$ increases, so does $\bar{M}_w^*$.

Fig.4 shows the boundaries of the regions of uniqueness (solid line) and stability (dashed line), at

$$\beta_1 = 0.5, \bar{M}_w = 0.1, \bar{T}_w = 5, Pr = 0.74, f_w = -2, \quad L = 0, \bar{C}_w = 0, E_w = 67.1366.$$

The size of the loop decreases with increasing $\bar{M}_w$, it moves into the region of nonuniqueness, and disappears altogether. For example, at $\bar{M}_w = 0.3$ the loop is absent. We see therefore that increasing the injection stabilizes the combustion. At strong injections the loss of stability and self-oscillations can occur at $L \to 0, E \to \infty$ only in the case when $\bar{M}_w < \bar{M}_w^*$ or when $\bar{T}_w > \bar{T}_w^*$ (at fixed $\bar{M}_w$). On the other hand, increasing the injection leads to substantial increase of the region of nonuniqueness. This follows e.g., from comparing the curve *1* of Fig.1 with the boundaries of the region of nonuniqueness at $f_w = -2$ when $L = 1$ (curve *2* of Fig.1) and $L = 0.01$ (curve *3* of Fig.1), with the remaining parameters assuming the values used in /10/. We see that for $\bar{C}_2 = 6.44518$ /10/ the value of $\bar{C}_{1,2}$ has increased compared with the case of $f_w = -0.5$, by four orders of magnitude, while the value of $\bar{C}_{1,1}$ decreased by two orders of magnitude. Such a sharp change in the values of $\bar{C}_{1,1}$ and $\bar{C}_{1,2}$ is caused by a considerable weakening of the influence of the wall on the combustion process when the injection is increased. Reduction in the value of L shifts the curve $\Delta_1 = 0$ into the region of smaller values of $\bar{C}_1$.

3. Assessment of the results. The results obtained yield the following conclusions.

1°. Strong injection stabilises the combustion, but does not eliminate the multiplicity of the stationary modes. Therefore, before carrying out the computations for a reactive boundary layer in the presence of strong injection in the traditional stationary formulation, the range of its applicability must first be estimated.

2^{o}. The fact that the results obtained agree with the values of $\bar{C}_{1,1}, \bar{C}_{1,2}$ computed in /9, 10/ to within the order of magnitude, enables us to expect that in the case of strong injection the system (1.1) will be even more suitable for carrying out an approximate qualitative analysis of the problem in question. In particular, (1.1) can be used to estimate the range of admissibility of the stationary formulation in the region of self-oscillations. The paper /15/ can serve as an example of successful matching a qualitative analysis with the numerical computations.

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